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State-Transition Structures in Physics and in Computation

C. A. Petri

Gesellschaft für Mathematik und Datenverarbeitung, mbH, Bonn, Federal Republic of Germany

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In order to establish close connections between physical and computational processes, it is assumed that the concepts of "state" and of "transition" are acceptable both to physicists and to computer scientists, at least in an informal way. The aim of this paper is to propose formal definitions of state and transition elements on the basis of very low level physical concepts in such a way that (1) all physically possible computations can be described as embedded in physical processes; (2) the computational aspects of physical processes can be described on a well-defined level of abstraction; (3) the gulf between the continuous models of physics and the discrete models of computer science can be bridged by simple mathematical constructs which may be given a physical interpretation; (4) a combinatorial, nonstatistical definition of "information" can be given on low levels of abstraction which may serve as a basis to derive higher-level concepts of information, e.g., by a statistical or probabilistic approach. Conceivable practical consequences are discussed.

INTRODUCTION

This paper attempts to provide a common basis for physical and computational ways of thinking. Our approach is motivated and guided by the practical advantages which may be gained by successfully doing so. If this approach should turn out to be a small, but definite step towards the remote (perhaps illusory) goal of founding technology and natural sciences on a theory of information flow, the author would feel rewarded beyond merit. He makes no claims whatever in this direction, but admits—in order to make his position clearer—that faint hopes for such a development have also influenced this approach to some extent.

We shall proceed as follows: We define four levels of description for processes and systems *below* the level of contemporary switching techniques,

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starting with the notion of a relativistic cotemporality relation between world points (level 0) and ending with the construction of reversible devices ("transfers") which are known to be sufficient for supporting all physically feasible computations (level 3). We shall not presuppose real-valued metrics, coordinate systems, rigid bodies, or precise clocks; rather, we shall rely on elementary topology and combinatorics, and prefer to work with binary relations. We expect that this self-imposed discipline will pay off in terms of simplicity, reliability, and range of application; e.g., an implementation of properly connected transfers should be self-synchronizing and require no tuning.

For the sake of brevity, we restrict our attention to physical effects of a single and very simple structural type: that of *interaction between two* (or more) *particles*. Two-particle interactions are *sufficient* to support well-organized information flow, including computation. However, in order to justify our line of reasoning, we shall have to build new bridges between the continuous models of physics and the "discrete" models of digital computing, and also to take a close look at the issue of partial orderings and disorders.

The numbered assumptions proposed in this paper do not refer to the microscopic scale only. Viewed as a collection of axioms, they have finite models.

STATES AND TRANSITIONS ON DIFFERENT LEVELS OF DESCRIPTION

We construct four conceptual levels (as in Brauer, 1980, p. 11) for the description of processes and systems, by successive abstraction. We assert that levels 0–2 can be used both in physics and in computing (including the behavior of distributed computer systems). Level 3 is an abstraction of level 2 which highlights the properties of information flow in the case of bitwise decomposition; devices with full computing capability have been specified on level 3. Individuals of level n will represent *sets* of individuals of level $(n-1)$.

Starting from level 2, physicists and computer scientists make different further abstractions in accordance with their different goals. It is suggested that both groups can benefit greatly by following a common path up to a point corresponding to our level 3, i.e., by agreeing on a common concept of information flow, at least on the lowest possible level.

Level 0: Concurrency Structure. Concurrency is short for "the binary relation of cotemporality of world points." Here we follow closely the

axiomatizations of relativistic space-time as proposed by Reichenbach, Lewin, and others since 1923, which are concisely compiled by Carnap (1958). For individuals, we take the time layers of *signals*, the smallest propagators of physical effects. Some signals are particles, others propagate from particle to particle through interaction. The history of each signal is a "world line" and consists of world points. Let x and y be individuals; we write $x < y$ iff $x \neq y$ and a signal passes from x to y. We define

$$x \,\mathrm{co}\, y : \Leftrightarrow \text{neither } x < y \text{ nor } y < x$$
$$x \,\mathrm{li}\, y : \Leftrightarrow x < y \text{ or } y < x \text{ or } x = y$$

$x \,\mathrm{li}\, y$ means that there is a (world) *line* which passes through x and through y. It follows that co and li are reflexive and symmetric, but not that they be transitive. We observe that

$$x \,\mathrm{co}\, y \text{ and } x \,\mathrm{li}\, y \Leftrightarrow x = y$$
$$x \,\mathrm{li}\, y \Leftrightarrow (x = y \text{ or not } x \,\mathrm{co}\, y)$$

or abbreviated: $\mathrm{co} \cap \mathrm{li} = \mathrm{id}$; $\mathrm{li} = \mathrm{id} \cup \overline{\mathrm{co}}$. Let us now forget that $<$ was originally introduced as a strict partial ordering (irreflexive and transitive), and concentrate on the properties of co, remembering that li and co are definable from each other. Let

$$\mathrm{Co}(x) := \{z \mid x \,\mathrm{co}\, z\}, \qquad \mathrm{Li}(x) := \{z \mid x \,\mathrm{li}\, z\}$$

If $\mathrm{Co}(x) = \mathrm{Co}(y)$ *or* $\mathrm{Li}(x) = \mathrm{Li}(y)$, we shall collect x and y into a cluster; such clusters are equivalence classes of world points, and will be the individuals of level 1.

Level 1: Occurrence nets (Brauer, 1980, p. 7 and p. 251ff). Consider a short piece of the history of a small number of particles which interact occasionally, pairwise (Figure 1). The graphical elements of this Figure will

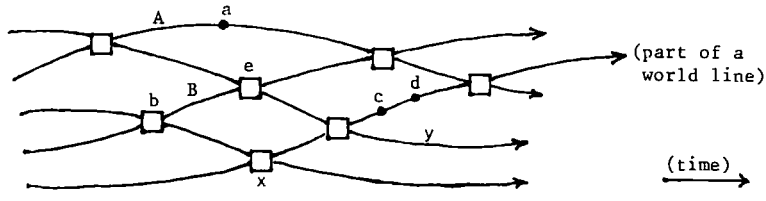


Fig. 1. A piece of the history of five interacting objects.

be called occurrences; some elements such as b denote interactions (occurrences of *transitions*); other elements such as A, B, a, b, x, y denote the prerequisites and results of interactions (occurrences of *states*). In Figure 1, we have $a\,\mathrm{co}\,b$, $A\,\mathrm{co}\,b$, $a\,\mathrm{co}\,x$, $b\,\mathrm{li}\,x$, $x\,\mathrm{li}\,y$, etc.

We shall now describe the structure of the set X of all occurrences, and its partition into a set S of state elements and a set T of transition elements. We do so by stating some assumptions (An) about the relation co in X, which is the image of (co on level 0).

(A1) $x \in X \Leftrightarrow x\,\mathrm{co}\,x$ (co is reflexive)
(A2) $x\,\mathrm{co}\,y \Leftrightarrow y\,\mathrm{co}\,x$ (co is symmetric)
(A3) $\mathrm{Co}(x) = \mathrm{Co}(y) \Leftrightarrow \mathrm{Li}(x) = \mathrm{Li}(y)$

A3 is not quite trivial, although it is equivalent to

$$\mathrm{Co}(x) = \mathrm{Co}(y) \Rightarrow x = y \quad \textit{and} \quad \mathrm{Li}(x) = \mathrm{Li}(y) \Rightarrow x = y$$

A3 means that level 1 structures can indeed arise by clustering of world points of level 0. This is not so for certain "degenerate" concurrency structures with infinitely many world points. (It is instructive to find an example.) For Figure 1, A3 implies that $c = d$, $A = a$, and that B consists of one occurrence only! Therefore $B \in X$; in terms of ordering, B is an *immediate successor* of b.

(A4) $|X| \geqslant 2$ (X contains at least two occurrences)

A4 appears to be harmless, as though it were introduced to exclude the formally cumbersome exception of a world without signals. However, in conjunction with A3, it implies that occurrences are not totally ordered (on a unique universal time scale), and moreover, that

$$\mathrm{co}^2 - \mathrm{co} \neq \varnothing \qquad (\textit{co is not transitive})$$

which holds for relativistic, but not for classical space-time.

We write $x\,\mathrm{co}^*\,y$ iff a finite sequence of co steps leads from x to y. We assume

(A5) $\mathrm{co}^* = \mathrm{li}^*$ [The structure (X, co) is "*coherent*"]

A5 implies that each pair x, y of occurrences is connected by a finite number of co steps, and also of li steps.

A subset $l \subset X$ will be called a *Line* iff it is a maximal set of occurrences which are pairwise in relation li.

A subset $c \subset X$ is called a *Cut* iff it is a maximal set of occurrences which are pairwise in relation co.

The old physical postulate (Carnap, 1958) that every Cut represents a spatial distribution (either in the spirit of $\mathbb{R}^3$, or of some phase space) can now be written as "every Cut cuts every Line":

(A6) Cut (c) and Line (l) $\Rightarrow c \cap l \neq \varnothing$ (co is "*K dense*")

which is a nontrivial, global assumption whose relevance to computing is being explored by Best (Brauer, 1980). *K* density implies a local property, called "*N* density" because the smallest possible structure which is not *K* dense can be depicted by an *N*-shaped diagram (on the left of Figure 2). The diagram on the left is *K* dense only if there exists an element z on a Line *between* x and y such that a Cut through a and b has an element (z) in common with that Line. The usual idea of "plain" density requires some z between every pair $x \neq y$ on a Line, which leads to an infinite number of intermediate points. *N* density requires some intermediate z for a well-defined purpose only, and does not lead to an infinite regress when $\mathrm{Li}(z) \subset \mathrm{Li}(x)$ and $\mathrm{Li}(z) \subset \mathrm{Li}(y)$. Therefore, it seems natural here to define a *proximity relation P* as follows:

$$xPy :\Leftrightarrow \mathrm{Li}(x) \subset \mathrm{Li}(y) \quad \text{and } x \neq y$$

xPy means that all Lines passing through x also pass through y. In the example of Figure 1, we have *BPb* and *BPe*. We call *B* a state occurrence (of some particle) beginning by the transition occurrence (interaction) *b* and ending by the transition occurrence *e*. Hence we can read xPy as "x is changed by y." In accordance with our interpretation, we shall assume for level 1:

(A7) $P^2 = \varnothing$ (there are no changes of changes)
(A8) For all $x, y \in X$: $x(P \cup P^{-1})^* y$

A8 is the assumption that (X, co) is a combinatorial structure, to which we

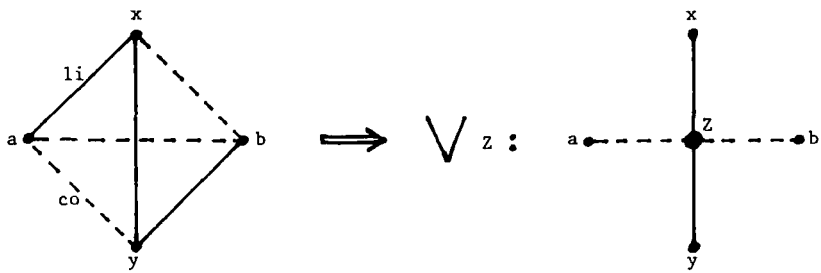


Fig. 2. Definition of *N* density.

assign the topology

$$a \subset X \quad \Rightarrow \quad \text{Closure (a)} := a \cup \{y \mid \vee x \in a : xPy\}$$

This is a "net topology," i.e., a topology with the property that every singleton is either open or closed, and that (not only every union but also) every intersection of open sets is open. Holt (1982) uses this topology and its dual (obtained by substituting "yPx" for "xPy" in the definition of closure) for a much deeper purpose; we have introduced it merely as a mathematical and conceptual tool for reaching all meaningful higher levels by applying *continuous mappings* (in the sense of topology) to level-1 structures. This tool will not be employed in this paper; for details, see Brauer (1980), p. 144ff. On the basis of A7 and A8, the set X can be partitioned into S and T:

$$S := \text{domain}\,(P) \qquad \text{the set of } \textit{state} \text{ elements}$$

$$T := \text{range}\,(P) \qquad \text{the set of } \textit{transition} \text{ elements}$$

$$S \cup T = X, \qquad S \cap T = \varnothing, \qquad P \subset S \times T, \qquad P \subset \text{li}$$

Now remember that co has been conceived originally as a *disorder* relation; we have to ask now, what properties co must have so that a consistent ordering on X exists, based on a local distinction between "past" and "future." The main difficulty here is to permit *finite* lines (which close to cycles of at least four elements) while still permitting the conventional infinite lines on which a total ordering exists.

Referring to Figure 1, we shall say that "b is immediately followed by B" and "B is immediately followed by e"; in shorthand: "bFB" and "BFe." We call a relation F between occurrences a *consistent orientation* on X iff:

$$\begin{aligned} F \cap F^{-1} = \varnothing \quad & \textit{and} \quad F \cup F^{-1} = P \cup P^{-1} \\ & \textit{and } F \circ F \subset \text{li} \qquad (\circ \text{ denotes relational product}) \\ & \textit{and } F \circ F^{-1} \subset \text{co} \quad \textit{and} \quad F^{-1} \circ F \subset \text{co} \end{aligned}$$

If there are no finite lines, the transitive closure of F is then a strict partial ordering on X, just as the relation "$<$" which we started with. We make three assumptions on ordering, without enquiring into their interdependence:

(A9) A consistent orientation F exists on X.

This is clearly a global assumption. It can be related to local assumptions by

defining the *proximity* of an occurrence x as

$$p(x) := \{y \mid xPy \quad or \quad yPx\}$$

(A10) Within every $p(x)$, $\quad co^2 \subseteq co \quad$ (co is *locally transitive*)

(A11) Within every $p(x)$, $\quad \varnothing \neq \overline{co}^2 \subseteq co \quad$ ($\overline{co}$ is locally orientable)

where $x \overline{co}\, y := \Leftrightarrow x \text{ li } y$ *and* $x \neq y$.

We call a structure (X, co) a *rope* (whose "threads" are the Lines defined above) when A1–A11 hold. A rope is a *straight rope* if it contains no finite lines (i.e., F cycles).

A rope, for which moreover A12 holds,

(A12) X is a finite set,

is called a *cyclic rope*.

The main consequences (Cn) of our assumptions are:

(C1) cyclic ropes exist: The rope "axioms" do have finite models. The smallest known rope has 12 elements (Figure 3) and has been associated with a quantum-mechanical harmonic oscillator at lowest energy level.

(C2) There are precisely two consistent orientations F_1, F_2 for each rope; $F_1 = F_2^{-1}$. Interchanging F_1 and F_2 corresponds to the formal operation of time reversal: $t \mapsto -t$.

(C3) Cyclic ropes can be represented by graphs, with vertices T, arcs S, and incidence relation F.

(C4) The triple (S, T, F) definable from each rope fulfills the axioms for *nets* (Brauer, 1980; Holt and Commoner, 1970; Petri, 1962), namely,

$$S \cap T = \varnothing\,;\; S \cup T \neq \varnothing\,;\; F \subseteq (S \times T) \cup (T \times S);\; \textit{and}$$

$$\text{domain}\,(F) \cup \text{range}(F) = S \cup T$$

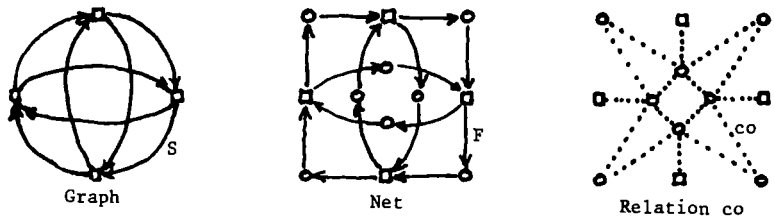


Fig. 3. Representations of smallest cyclic rope.

Ropes correspond to special kinds of nets: We get

1. $F \cap F^{-1} = \varnothing$ (rope nets are "pure" nets), and
2. Every S element has exactly one immediate predecessor and exactly one immediate successor; i.e., every state occurrence has one beginning and one ending, in contrast to level-2 structures. Rope nets are "synchronization nets" and, if finite, "synchronization graphs" (Brauer, 1980, p. 530). Such graphs are often used in distributed computing.

(C5) Straight ropes have a generalized Dedekind continuity, the kind of continuity found in the total ordering of real numbers, but generalized for partial orders. Those Cuts which contain S elements only play the role of real numbers. For details, see Brauer (1980), p. 257ff.

(C6) The arcs of Feynman graphs may be viewed as S elements of a rope, their vertices as T elements. Some of our assumptions do not hold in Feynman graphs, for two reasons:

1. A Feynman graph is not intended to describe the *full* history of *all* particles;
2. A more refined definition of the proximity relation P would be needed to take into account the qualitative differences of particles, which we have not considered. With P as defined in this paper, A7 does not permit a spontaneous decay of unstable particles.

Level 2: Condition–Event Systems (Brauer, 1980; Holt and Commoner, 1970). They are obtained by folding together a level-1 structure, *or part of it*, in such a way that, e.g., the occurrences "summer 1979," "summer 1980," "summer 1981" are mapped onto a single S element "summer" of level 2. We call such an S element a *condition* and not a state, because we do not want to refer to the object which is in that state. Indeed, this object might be a different one for each of the repeated occurrences (level 1) of a condition (level 2); usually a different signal, but also even a different microscopic or macroscopic particle. Likewise, we use the term *event* for "the image of a set of transition occurrences." Using the description in (S, T, F) form both on level 1 and level 2, the folding must be done in such a way that the relations P and F (proximity and direction of flow) are preserved. The resulting level-2 net must be *pure* ($F \cap F^{-1} = \varnothing$, i.e., a condition cannot end and begin simultaneously. The more regular the behavior history (Level 1) of a set of particles is, the smaller can the resulting net be made. The appropriate folding of the smallest cyclic rope is an identity map; folding applied to the infinite net of Figure 4 yields a net isomorphic to that of Figure 3.

The important *structural* difference between level-1 nets and level-2 nets is that on level 2, S elements (conditions) can have *more than one* immediate predecessor and successor. By avoiding a too-tight folding, we

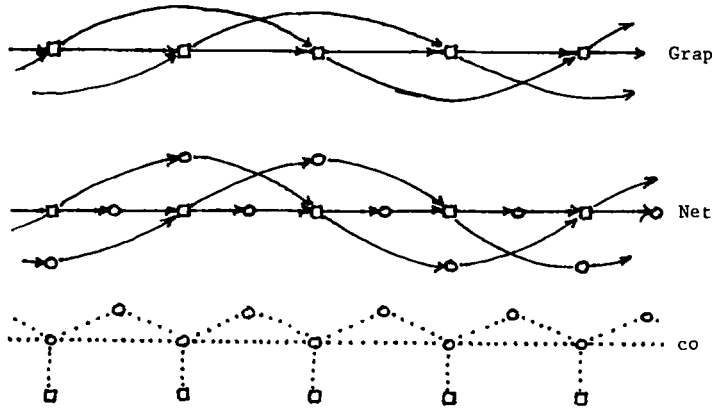


Fig. 4. Partial representations of the rope obtained by "straightening" the rope of Figure 3.

can ensure that each condition has *no more than two* such "input events," or, respectively, "output events," as we shall call them. To indicate concurrent presence, we use black dots (pebbles, tokens) on the elements in the graphical representation, to denote the image of some *Cut* as defined above. A level-2 net can be thought of as a "distributed automaton" (a normal automaton could carry one pebble only); it can also be thought of as a physical system of interacting particles (and/or signals). The behavior of the system can be simulated by "pebbling" or the "token game," a widespread custom in the computer field. The rules of those games should be founded on level-1 concepts; if they are, every simulation yields a *unique* level-1 structure, except in situations of "*conflict*" (Figure 5). Suppose that condition y is currently holding (the condition symbol $\bigcirc y$ carries a pebble) and has two output events i and k. Which of them will subsequently have an occurrence? We cannot tell; we have too little information. We only know, from our considerations on level 1, that i and k cannot have *concurrent* occurrences, since their proximities overlap (a pebble on a condition "cannot be split apart"). The conflict can only be resolved if we can look at *all the preconditions* of i and k. Assume that x, y, and z are all of these. Now if x, y, and z are all holding concurrently, the situation would still be a conflict. But the system whose description is in Fig. 5 would "stop behaving" if the conflict is not resolved! Therefore we shall put down our final assumption:

(A13) Conflicts in condition–event systems are resolved.

This leaves open the question of *how* they are resolved. Before the resolution, the *information* on "whether i or k" is not contained in the system

Fig. 5. Conflict situations.

description (including the position of pebbles); after the resolution, it is. By the very act of deciding the conflict between i and k, we shall define, *one bit of information* has come into the system. From where? Not out of the system, anyway. Therefore: from its environment, from the rest of the universe, if the question "from where" is to have any meaning at all. And if the system has no environment? Then it must be a (the) universe itself, closed with respect to information flow. If we care to follow this usage of words, we can reformulate:

(A13.1) A detailed level-2 description of the universe contains no conflict situations.

This has been called the assumption (axiom) of "local determinacy" (Brauer, 1980).

Remark: The time reversal invariance implied by A1–A11 yields in conjunction with, and applied to, A13 a "law of conservation of information," the starting point of Kantor's "information mechanics." This law is equivalent to the assumption that the *information flux lines* of our level 3, which we are about to construct now, have no beginning and no ending.

Level 3: Information Flow Graphs. These are intended to describe the flow of information (as opposed to the "flow of pebbles" on level 2!) in the greatest possible detail, i.e., in bitwise decomposition. Higher levels can be easily reached from level 3 by following well-established lines of thinking.

Let us take another look at Figure 5 in the light of A13.1. Let us assume that $\{x, y, z\}$ is the *full* set of preconditions of the events i and k—in the universe, not only in the system of particles under consideration. Then, by A13.1, since y holds, x and z cannot hold concurrently! The conflict situation on the left side of Figure 5 can be resolved only on the basis of the *distribution of one single holding* (pebble) over the set $\{x, z\}$ of conditions. Under our assumptions, the distribution shown on the right side of Figure 5 cannot exist We now declare the set $\{x, z\}$ of level-2 elements to be an individual of level 3. In general: we declare every pair $\{u, v\}$ of conditions which are linked by a chain of four F arrows of alternating direction, to be an individual of level 3, an individual of type S, like a state occurrence and like a condition. Further, we cluster all events of level 2 in

such a way that each pair $\{u, v\}$ has at most one cluster of input events and at most one cluster of output events. We do not cluster unless it is demanded by these rules. We declare the clusters of events to be individuals of level 3 of type T and call these individuals *transfers*. The level-3 individuals of type S we call *stations*. We have produced a net (S, T, F) of stations and transfers. By pebbling on a graphical representation of this net, we can observe bits of information flowing from stations to stations, interacting with each other in a *reversible* way while passing through transfers.

But hold on! Our construction has not yet been tested as to whether it has a physical meaning. The careful reader will have noticed that a given condition u might have no partner v as required to establish a station for one bit; or u might belong to a station $\{u, v\}$ and also to another station $\{u, w\}$. In the same sense, transfers might overlap, etc. Our answer to this dilemma is: The given construction of level 3 has a physical meaning only if it is, mathematically, a (continuous) *function* from level-2 nets onto level-3 nets. For most of the numerous condition–event systems we find in the literature under various names, no such function exists, for the simple reason that their purpose was not related to *bitwise* decomposition of physical information flow. On the other hand, we can decompose every information flow graph into a condition–event net. It remains to convey a feeling for the structure and usage of information flow graphs, by employing pictorial notations. For details, we must refer to Brauer (1980), Petri (1967), and Shaw (1977). (Several blueprints for full-fledged computers exist in a graphical level-3 notation.) The main differences from conventional switching logic are the following:

1. By the occurrence of a transfer, its output information appears *and its input information disappears*.
2. The input information can be recomputed from the output information; transfers are *reversible*.

The transfers in Figure 6 are "P1 transfers" only. Their definition in terms of level-2 elements is given in Figure 7. Such transfers are not sufficient to encompass all Boolean functions needed for computation. Two more types

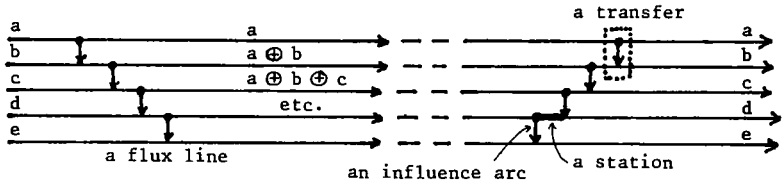
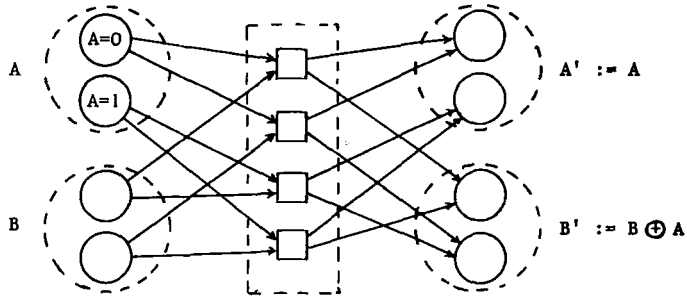


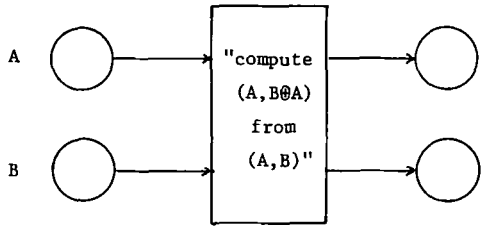
Fig. 6. An information flowgraph. Left: Encoding of binary numbers into Gray code. Right: Decoding.

Level 2 :

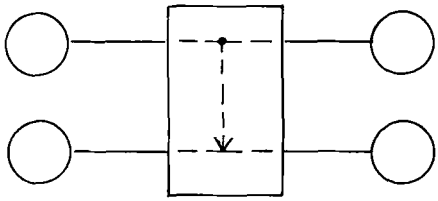


(folding into Level 3 is indicated by dotted lines)

Level 3 :



New notation for activity defined above :



Abstract notation for information flow graph :

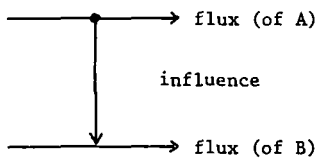


Fig. 7. Construction of a P1 transfer.

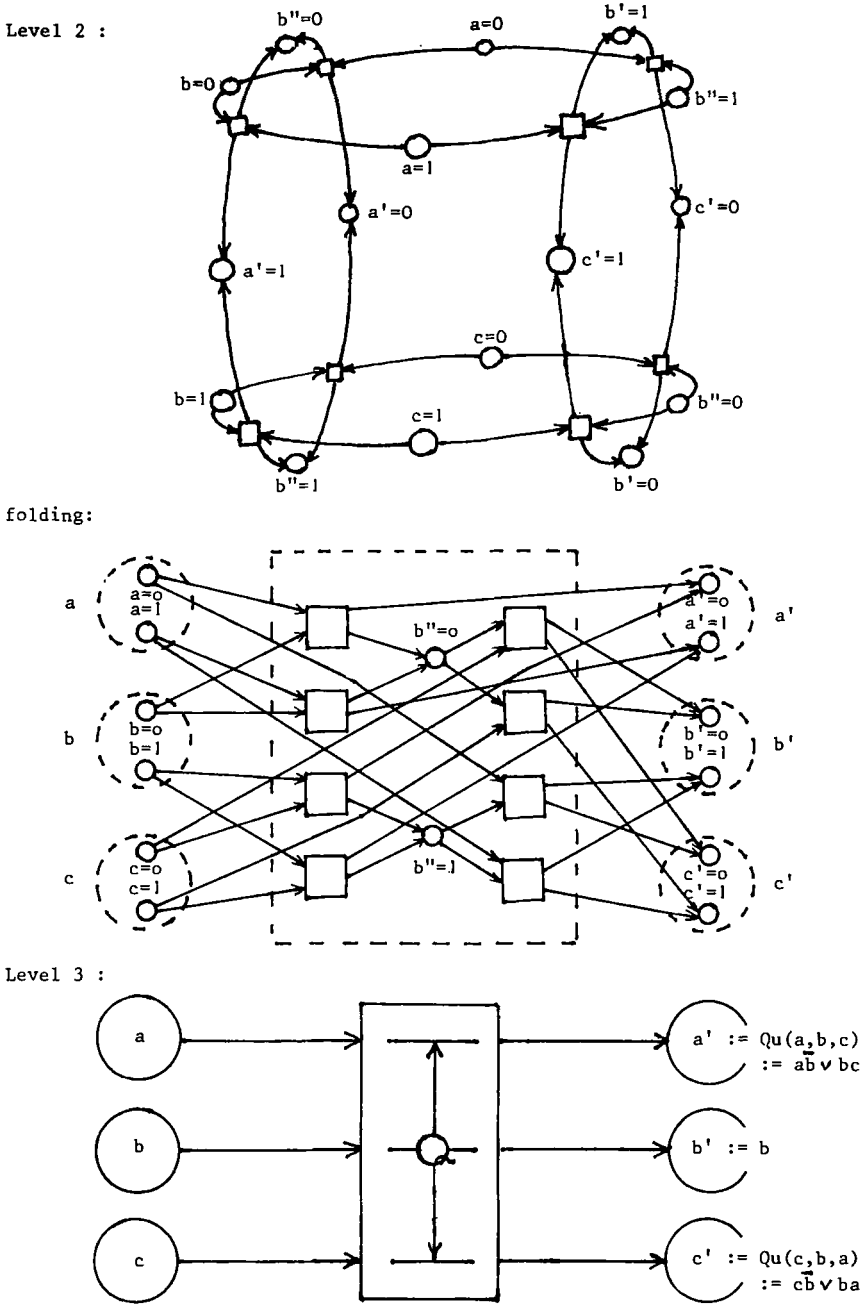


Fig. 8. Construction of a Q transfer.

of entities are required for that purpose:

1. A transfer with three inputs and three outputs which embeds a logical AND reversibly in a Boolean bijection. We choose the "Quine transfer" (Q transfer) for this purpose, because of its highly symmetrical level-2 decomposition: Figure 8.
2. A set of stations such that whenever they contain a bit, that bit represents the same Boolean value. Such stations might be called "sources of a constant bit stream." Now a constant bit stream does not transport information in the usual sense. We need a new concept here: we call the bits originating from a constant source bits of *enlogy*, as opposed to bits of information. Enlogy is needed, e.g., for copying information.

CONCLUDING REMARKS ON EXPERIENCE

After a certain amount of exercise, it becomes quite easy to produce logical designs of considerable size in terms of information flow graphs, mainly, in the beginning, by a modular approach. These designs get a quite new quality when one treats *enlogy as a scarce resource* and demands economical use of enlogy. One tries to construct, in addition to the desired outputs, *enlogy sinks* at the expense of, in the worst case, doubling the size of the design. Thus, enlogy can be *recycled* by identifying sinks and sources of enlogy. In a physical implementation, this means avoiding dissipation, or degradation, of energy.

Entropy has been studied in terms of enlogy loss.

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